Inventory and Implications for Valuation

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Outline

- Main Themes
- Pricing and Delivery
- Inventory and Forward Yields
- Introduction to Storage Valuation
- CSOs
- Volatility Term Structure

Main Themes

- Liquidity Versus Dimensionality
  - Liquidity is concentrated in benchmarks.
  - Risk can be spread over hundreds of locations and many delivery months.
  - Price decoupling (specialness) occurs routinely.
- Covariance Structure Versus Options Markets
  - The term structure of volatility and correlation is non-trivial.
  - Liquidity in options is concentrated in vanilla products with mechanics which limit information about (limit utility as hedges of) covariance risk.
  - Dependence on models as price extrapolation devices occurs even for "simple" structures.

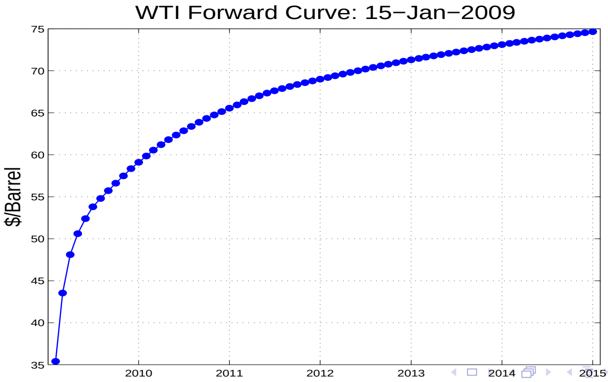
Main Themes

- Broad Range of Time-Scales
  - Price dynamics exhibits structure over years ("infrared") and down to days and hours ("ultraviolet").
  - Liquidity in daily tradables, both forwards and options, limit one's ability to calibrate models and hedge risk.
  - This is particularly true for commodities with rapid fluctuations in demand and limited "storability" (e.g. natural gas and power).
- Uncommoditized Risks
  - Many structures involve risks which are effectively untraded.
  - These include:
    - Variable quantity swaps arising from demand and production hedges.
    - Structures and assets with flexibility at the hourly time-scale.
    - Commodity-linked credit structures for many "names".
  - Standard risk-neutral methods are of questionable value.

Pricing and Delivery

Forward Pricing and Delivery

- This figure shows the forward curve for WTI on 15Jan2009.
- Each point represents the price for WTI delivered in subsequent months as of this pricing date.
- We will denote the forward price observed at time t for delivery at time T by $F(t, T)$.



Pricing and Delivery

Examples

- WTI Crude CME/NYMEX (Futures)
  - Notional 1000 barrels delivered anytime in the contract month
  - Specific grades of crude (adjusted for value) delivered at Cushing, OK
- Natural Gas CME/NYMEX (Futures)
  - Notional 10,000 MMBtus delivered ratably over the contract month
  - Delivery location: Henry Hub, LA

Pricing and Delivery

Examples

- Gas Daily Swap (OTC/Cleared)
  - Buyer will pay seller a fixed price per MMBtu for 10,000 MMBtu's per day of natural gas in Dec 2011
  - Seller will pay buyer the average Gas Daily Index at Henry Hub for the delivery month.
    - Gas Daily Index prices are published by Platts for many delivery locations.
    - At each locations the GDI a volumetric weighted average of a survey of trades done for physical delivery.
    - Survey prices at highly liquid locations are more reliable than at illiquid locations.
  - Settlement is 10 business days after the last flow date.

Pricing and Delivery

Examples

- Natural Gas Penultimate Swaps
  - Settle on the futures settlement price on the next-to-last trading day of the analogous futures contract.
  - Options on natural gas futures expire on the same day, rendering this swap particularly useful for options hedging.
  - The settlement amount is based on the difference between the futures penultimate settlement and the strike at which the trade was done.

Pricing and Delivery

Options

- Mechanics varies by commodity.
- Common Features:
  - Options mechanics tend to mirror conventions for futures and swaps.
  - Expiration can result in either financial settlement or physical positions.
  - Expiry is usually "close" to the contract month.
    - "MxN" options markets where expiry can be M units of time before delivery at N are not traded.
  - Multiple Time Scales
    - Typically markets support options that exercise into monthly exposure or into annual (cal strip) exposure.
    - For power daily options are commonly traded.

Pricing and Delivery

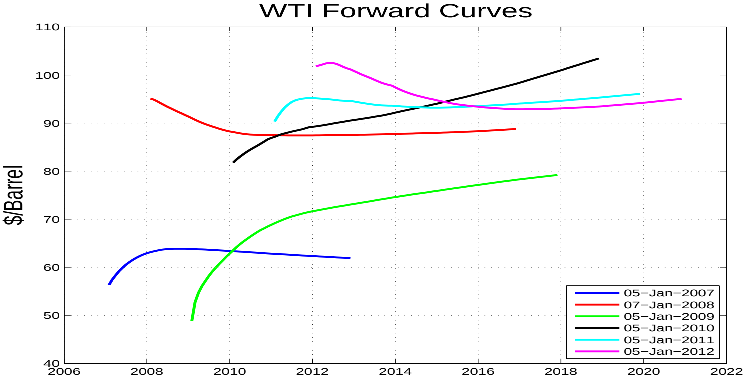
Spot Prices

- The floating price in the last example is often referred to a "spot price", which is the price for "immediate" delivery.
- A spot price is in almost all situations technically a forward price with a delivery time very close to the present.
  - Formally the spot prices is represented by $F(t, t)$
- In practice, the price is usually established slightly before the delivery time, rendering the distinction between spot and forward somewhat arbitrary.
  - In the case of natural gas, trading for delivery on day d occurs on day $d - 1$ which is when the index print is established.
  - For power the spot price can be set a day before, hour before or immediately at delivery.
  - For coal in which logistics and shipment are an issue, "spot" can refer to a time-lag between trade date and delivery measured in weeks or months.

Inventory and Forward Yields

Snapshots

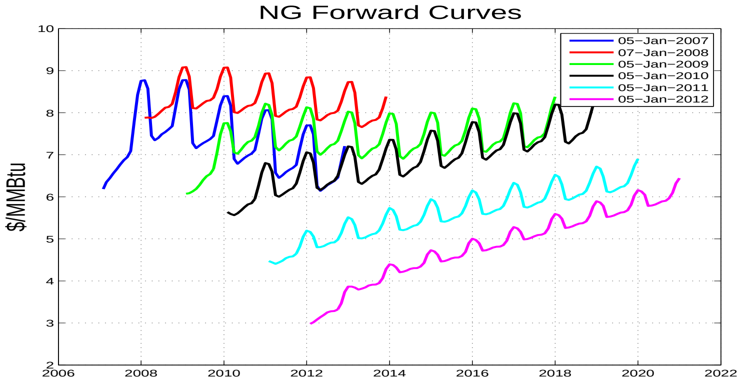
- WTI forward curve at a variety of dates:
  - Note the range of prices as well as the changes in the monotonicity



Inventory and Forward Yields

Snapshots

- NG forward curve at a variety of dates:
- Note:
  - The seasonality superimposed on macro trends.
  - The breakdown from the WTI price levels in recent years.



Inventory and Forward Yields

The Carry Formalism

- Forward curves can be viewed as yield curves.
- Forward yield:

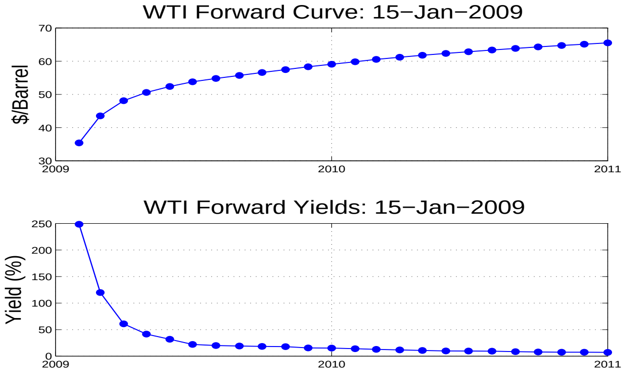
$$y(t, T, T+S) = \frac{1}{S} \log \left[\frac{F(t, T+S)}{F(t, T)} \right]$$

- The forward yield annualized rate implied by borrowing to buy the commodity at time T and sell it at time $T + S$.
- Negative forward yields imply that market participants are willing to pay a premium for earlier delivery
  - This is effectively lending at negative rates.
  - This happens when supply is contrained.

Inventory and Forward Yields

The Carry Formalism

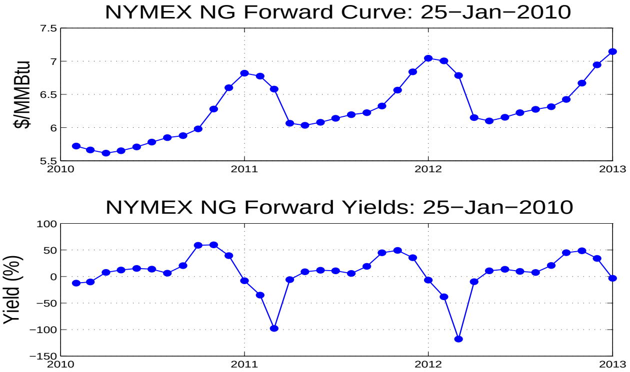
- Yields often exhibit extreme values.
  - The following is the WTI forward curve and forward yield for $S =$ one month in early Jan2009.



Inventory and Forward Yields

The Carry Formalism

- Seasonality yields negative forward yields consistently for seasonal commodities.



Inventory and Forward Yields

The Carry Formalism

- For a consumption commodity all we can state with certainty is that:

$$F(t,T) \leq F(t,t)e^{[r(t,T)+q(t,T)](T-t)}.$$

  - One can always buy the commodity at the spot price and ensure storage to delivery at T.
  - The convenience yield provides the comfort of an seeing an equality:

$$F(t,T) = F(t,t)e^{[r(t,T)+q(t,T)-\eta(t,T)](T-t)}.$$

- Key Points:
  - All that can be ascertained from market data is $q - \eta$
  - The cost of storage is not exogenous.
    * Storage owners will charge what the market will bear
    * The cost of storage is in reality a function of forwards and vols as opposed to an input.

Inventory and Forward Yields

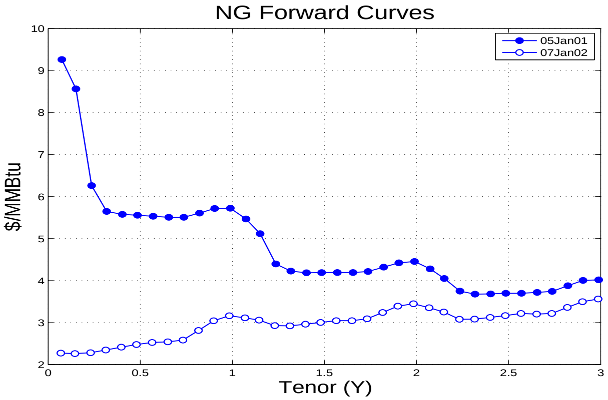
The Carry Formalism

- Incentives: The huge credit-crisis contango resulted in a massive increase in the use of VLCCs store oil and refined products.
- The figure shows the result outside of the Port of Singapore during Jan2009. (Source: Google Maps)

Inventory and Forward Yields

The Role of Storage

- Forward curves on January 2001 and January 2002.
  - Note the higher prices, backwardation and greater seasonality in 2001.



Inventory and Forward Yields

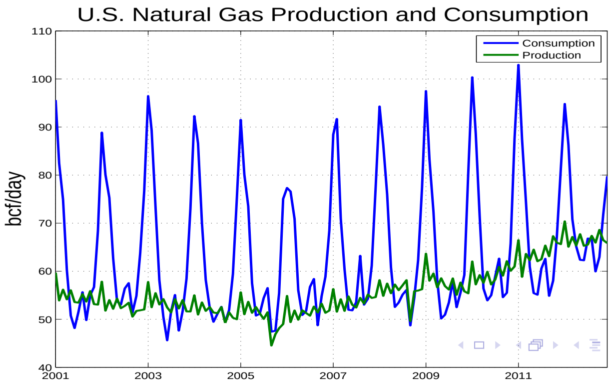
The Role of Storage

- Facilities exist to store commodities surplus quantities to accomodate anticipated (seasonal) and unanticipated demand fluctuations.
- Natural gas is particularly interesting.
- U.S. Natural Gas Markets
  - Annual gas consumption is roughly 25 Tcf with roughly 4 Tcf of imports.
  - Gas consumption is highly seasonal due to winter heating requirements.
  - Approximately 4 Tcf of natural gas storage facilitates accommodation of winter demand.

Inventory and Forward Yields

The Role of Storage

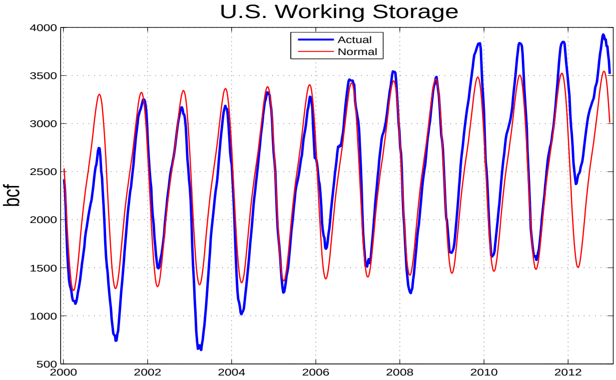
- North American demand is highly seasonal.
- Observations:
  - High winter peak demand and the relatively mild summer peaks (air conditioning power demand met with CC generation)
  - Non-seasonal production profile.
  - Recent increase in domestic production—shale gas glut.



Inventory and Forward Yields

The Role of Storage

- Roughly 4.2bcf of storage capacity resolves the production/consumption mismatch.
- Compare historical inventory levels versus "normal"
  - "Normal" is a Fourier fit with the number of modes used determined by an out-of-sample selection method with estimates of working capacity.



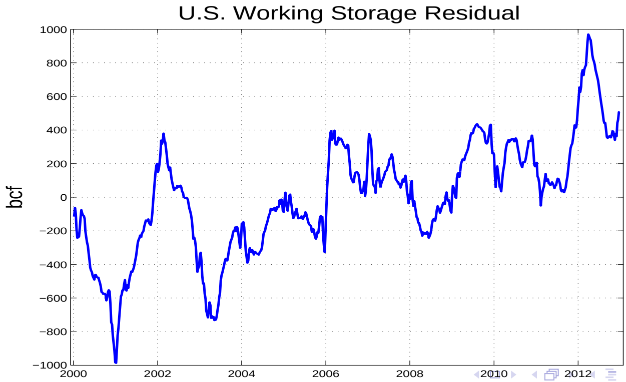
Inventory and Forward Yields

The Role of Storage

- Storage Residual: $R(t) \equiv S(t) - \bar{S}(t)$

$$\bar{S}(t) = \alpha + \beta t + \sum_{k=1}^{K} [\gamma_k \sin(2\pi k t) + \delta_k \cos(2\pi k t)] \quad (1)$$

- Compare Jan2001 to Jan2002 (the dates of the aforementioned forward curves).



Introduction to Storage Valuation

Types of Storage

- Types of storage:
  - Aquifer storage—facilities consisting of large porous rock structures.
  - Reservoir storage—depleted natural gas or oil production fields suitably modified for natural gas storage.
  - Salt caverns—salt dome formations that can be engineered to store natural gas.
- There is actually a total of roughly 8.7 tcf of natural gas storage.
  - Of this roughly 4.5 tcf is "base gas"—gas that must be injected once and is required to maintain working pressure.
  - The remaining inventory is referred to as "working storage".

Introduction to Storage Valuation

Constraints

- Operational constraints:
  - Capacity constraints: Ensure that enough capacity exists to accommodate a storage contract.
    - This involves specifying bounds on contracted capacity: $S(t) \in [0, S_{\max}]$ where $S(t)$ denotes the inventory level at time t.
  - Rate constraints: Ensure that contracted injection and withdrawal rates can be achieved.
    - This is accomplished by constraints on the rates: $s(t) \in [s_*, s^*]$ where $s(t) \equiv \frac{dS}{dt}$ is the injection rate with e $s_* < 0$ and $s^* > 0$.
    - In practice, as inventory approaches the bounds 0 or $S_{\max}$, these constraints can change—as the tank gets full (empty) putting more in (taking more out) becomes harder. Such constraints take the form: $s_* = s_*(S(t))$ and $s^* = s^*(S(t))$.

Introduction to Storage Valuation

Constraints

- Operational constraints:
  - Inventory constraints: All storage contracts specify initial and final conditions.
    - The most common structure: $S(T) = S(0)$ where T denotes the term of the contract.
    - Often the contract spans the start of the injection season 01Apr and terminates at the end of the withdrawal season 31Mar of the following year.
    - Some structures constrain inventory to specified inequalities at intermediate times: $S_{\vec{\tau}_*} \leq \vec{L}_*$ and $S_{\vec{\tau}^*} \geq \vec{L}^*$ for specific times $\vec{\tau}$ and limits $\vec{L}$.

Introduction to Storage Valuation

Constraints

- Cyclability:
  - The number of times it can fill and then empty, in the parlance "cycle" or "turn", in one year.
    - In the case of constant limits, one cycle time is:

$$\tau = \frac{S_{\max}}{|s_*|} + \frac{S_{\max}}{s^*}$$

    - The number of turns is $1/\tau$.
  - Most aquifer and reservoir storage is effectively "one turn storage", that is one cycle per year.
  - Salt cavern storage is much more flexible with some facilities able to turn several times a year.

Introduction to Storage Valuation

Storage: Dynamic Optimization

- Actual storage affords the owner the option of injecting and withdrawing on a daily basis.
  - Each decision effects the future optionality due to effects in existing inventory.
- The universal valuation constraint is that inventory $S(t)$ must satisfy: $0 \leq S(t) \leq S_{max}$.
- Other commonly encountered constraints:
  - Injection/withdrawal rate constraints: $s_* \leq S'(t) \leq s^*$.
  - s_* and $s*$ are functions of inventory level $S(t)$.
  - Hysteresis: s_* and $s*$ are functions of $[S(u) : u \leq t]$.
  - Ratchets: $S(\bar{\tau}) >= \bar{S}_{level}$ and similar upper bounds.

Introduction to Storage Valuation

Storage: Dynamic Optimization

- Formally, the valuation problem is:

$$V[0, S(0), F(0,\cdot)] = \sup_{s(\cdot)\in\mathcal{A}} \tilde{E}\left\{\int_0^{T_*} d(0,t)[-s(t)F(t,t) - \kappa(s(t), S(t), F(t,t))]dt\right\}$$

where:

- $d()$ and $F()$ are the discount factor and forward curve respectively.
- $S(t)$ is the current inventory level and $s(t) = S'(t)$.
- κ denotes costs associated with injection and withdrawal. For example, a cost that is a fraction of the fuel charge would take the form $\kappa = k|s(t)|F(t,t)$.
- $\mathcal{A}$ denotes allowed controls (for example: $0 \leq S(t) \leq S_{\max}$).

- This is a very hard problem.

Introduction to Storage Valuation

CSOs

- The simplest storage structure is a modified CSO.
  - Consider options with the following payoff:

$$\max\left[F(\tau, T+U) - e^{rU}F(\tau, T), 0\right]$$

    where we are assuming constant interest rates.
  - The owner of this option has the ability to:
    - Purchase natural gas at exercise time τ for delivery at time T at price $F(\tau, T)$;
    - Funding the cost to time $T+U$;
    - Selling the same quantity forward at time $T+U$.

Introduction to Storage Valuation

CSOs

- Exercise of CSO:
  - The holder of such an option will only exercise if the accrued cost of the purchase is less than the forward price at withdrawal

$$y(\tau, T, T+U) = \frac{1}{U} \log \left[\frac{F(\tau, T+U)}{F(\tau, T)} \right] > r$$

- Storage value is driven by the difference between forward yields and financing costs.

Introduction to Storage Valuation

Virtual Storage

- Consider a storage facility where the only constraint is $S(t) \in [0, S_{\max}]$.
  - There are no constraints on rates.
  - This type of structure is sometimes referred to as virtual storage and is clearly an idealization—the owner can toggle between empty and full instantaneously.
  - One purpose for considering virtual storage is that it is analytically tractable.
  - The value function does not depend on S_t since this can be changed instantly.

Introduction to Storage Valuation

Virtual Storage

- In discrete time virtual storage is the sum of nearest-neighbor CSOs:

$$V\left[T_n, F(T_n, \cdot)\right] = S_{\max} \tilde{E}_{T_n}\left[\sum_{m=n}^{N_*} e^{(T_m - T_n)r} \max\left[F(T_m, T_{m+1}) - e^{r\Delta t} F(T_m, T_m), 0\right]\right]$$

  where the problem is defined on the time grid $T \in \{n\Delta T\}_{n=1}^{N_*}$.

- Evaluation of virtual storage is equivalent to CSO valuation.

Introduction to Storage Valuation

Addendum on Gaussian Exponential Models

- Gaussian exponential framework:

$$dF(t,T) = F(t,T) \sum_{j=1}^{J} \sigma_j(T) e^{-\beta_j(T-t)} dB_t^{(j)}$$

  - We have for now the form $\sigma_j(T)$.
  - Often the BMs are assumed independent for simplicity.

- Intuition (2-factor):
  - If $\sigma_2 \equiv 0$, this is a one-factor model identical to that described in the first section:

$$\sigma(T-t) = \alpha e^{-\beta(T-t)}$$

  - The second factor will typically have $\beta_2 \ggg \beta_1$ and is intended to represent shorter time-scale forward returns.

Introduction to Storage Valuation

Addendum on Gaussian Exponential Models

- This is "HJM" for commodities introduced by Clewlow-Strickland.
- Some useful facts:
  - The integral of the returns for factor j on contract T over $[0, t]$ is:

$$\begin{aligned}\sigma_j(T)\int_0^t e^{-\beta_j(T-s)}dB_s^{(j)} &= \sigma_j(T)e^{-\beta_j(T-t)}\int_0^t e^{-\beta_j(t-s)}dB_s^{(j)} \\ &= \sigma_j(T)e^{-\beta_j(T-t)}Y_j(s)\end{aligned}$$

where we have defined: $Y_j(t) = \int_0^t e^{-\beta_j(t-s)}dB_s^{(j)}$.

*****This means that the distribution of returns for all tenors are simultaneously described by the processes $Y_j(t)$.**

*****This means that the dynamics of the entire forward curve are prescribe by a J-dimensional stochastic process.**

Introduction to Storage Valuation

Addendum on Gaussian Exponential Models

- Some useful facts: (cont)
  - **Returns are normally distributed** since any integral of the form $\int_0^t \phi(u)dB_u$ is normally distributed with mean zero.

    The variance is obtained by the Ito isometry:

$$E\left[\left(\int_0^t \phi(s)dB_s\right)^2\right] = \int_0^t \phi^2(s)ds.$$

    Therefore, the returns variance for $F(t, T)$ is:

$$V(t, T) \equiv \sum_{j=1}^{J} \sigma_j^2(T) \int_0^t e^{-2\beta_j(T-s)}ds$$

Introduction to Storage Valuation

Addendum on Gaussian Exponential Models

- Some useful facts: (cont)

  - Recalling that $Y_j(t) = \int_0^t e^{-\beta_j(t-s)} dB_s^{(j)}$, differentiating with respect to t yields:

    $$dY_j = -\beta_j \left[\int_0^t e^{-\beta_j(t-s)} dB_s^{(j)} \right] dt + dB_t^{(j)}$$

    or

    $$dY_j = -\beta_j Y_j dt + dB_t^{(j)}$$

    The Y's mean-revert toward a mean of zero with mean-reversion rates specified by the β's.

    Diffusions of this form are called Ornstein-Uhlenbeck processes.

    Properties:

    - $E[Y_t | Y_0] = Y_0 e^{-\beta t}$
    - $\mathrm{var}[Y_t | Y_0] = \frac{1-e^{-2\beta t}}{2\beta}$
    - $\mathrm{var}[Y_\infty] = \frac{1}{2\beta}$

Addendum on Gaussian Exponential Models

- Some useful facts: (cont)
  - The resulting form for $F(t, T)$ is:

$$F(t,T) = F(0,T) e^{\sum_{j=1}^{J} \left[\sigma_j(T) e^{-\beta_j (T-t)} Y_j(t) \right] - \frac{1}{2} V(t,T)}$$

  - To see this note that $F(t, T)$ is a martingale and that for any normal random variable Z: $E\left[e^Z\right] = e^{\frac{1}{2}\sigma_Z^2}$.
  - Note that the previous calculation is nothing more than the exponential equivalent of the previous GBM integration:

$$F_t = F_0 \ e^{-\frac{1}{2}\sigma^2 t + \sigma B_t}$$

with $\sigma^2 t$ replaced by the appropriate exponential integrals.

Introduction to Storage Valuation

Valuation

- Valuation proceeds in the usual CSO fashion:
  - The m^{th} option value is:

$$V_{n,m} = e^{-(T_m - T_n)r} \left[F(T_n, T_{m+1}) \Phi(d_1) - e^{r\Delta T} F(T_n, T_m) \Phi(d_2) \right]$$

where

$$d_{1,2} = \frac{\log \left[\frac{F(T_n, T_{m+1})}{e^{r\Delta T} F(T_n, T_m)} \pm \frac{1}{2} \tilde{\sigma}^2 \right]}{\tilde{\sigma}}$$

where:

$$\tilde{\sigma}^2 = (T_m - T_n) \left[\bar{\sigma}^2_{m,m+1} - 2\rho_{m,m+1} \bar{\sigma}^2_{m,m} \bar{\sigma}^2_{m,m+1} + \bar{\sigma}^2_{m,m} \right]$$

  - $\bar{\sigma}^2_{m,m}$ and $\bar{\sigma}^2_{m,m+1}$ are the term volatilities of contracts T_m and T_{m+1} over the time interval $[0, T_m]$ respectively;
  - $\rho_{m,m+1}$ is the term correlation between the two contracts over the same interval.

Introduction to Storage Valuation

Valuation

- Taking the continuous time limit ($\Delta T \to 0$) yields:

$$V(t, \vec{Y}_t) = \int_t^{T_*} e^{-r(T-t)} F(t,T) v(t,T) \left[h(t,T)\Phi(h(t,T)) + \phi(h(t,T)) \right] dT$$

where:

- The variance term is:

$$v^2(t,T) = \sum_j \left[\beta_j - \frac{\sigma_j'}{\sigma_j}(T) \right]^2 \frac{\sigma_j^2}{2\beta_j} \left[1 - e^{-2\beta_j(T-t)} \right]$$

- The "carry" term is:

$$h(t,T) = \frac{1}{v(t,T)} \left[\frac{\partial \log F(t,U)}{\partial U} \bigg|_{U=T} - r \right]$$

- Note that $\vec{Y}_t$ are the state-variables—the OU processes driving the forward curve.

Introduction to Storage Valuation

Valuation

- The previous formula is the closest to basic Black that you will find in storage valuation.
- The valuation formula permits fast exploration of basic attributes.
- Example: Intrinsic value:
  - Intrinsic value is generally not simple to calculate.
  - Here the result can obtained directly.
  - The optimal inventory path in the zero-vol case is:

$$S_{\mathrm{Intrinsic}}(t, T) = \begin{cases} S_{\max} & \text{if } \left.\frac{\partial \log F(t,U)}{\partial U}\right|_{U=T} - r > 0 \\ 0 & \text{otherwise} \end{cases}$$

  - This yields an intrinsic value of:

$$\int_t^{T_*} S_{\max} \left[\left.\frac{\partial \log F(t, U)}{\partial U}\right|_{U=T} - r \right]^+ dT$$

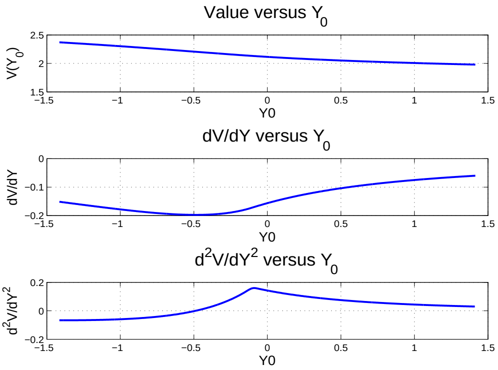
Introduction to Storage Valuation

Valuation

- Simple case in which $V(t, \bar{Y}_t)$ does not depend on t.
  - $F(0, T) = \text{constant}$
  - $T_* = \infty$
- The next slide shows results in the one-factor setting:
  - The top figure is the value versus $Y(0)$.
  - Negative values of $Y(0)$ correspond to contango, therefore, the value function must be a decreasing function of $Y(0)$ in order to be an increasing function of the forward yield.
  - The middle figure shows the change in value with respect to $Y(0)$; as this is a single factor model the plot shows $\Delta_Y = \frac{\partial V}{\partial Y}$ versus $Y(0)$.
  - The bottom plot of $\Gamma = \frac{\partial^2 V}{\partial Y^2}$ versus $Y(0)$ illustrates an important point: as with vanilla options, the convexity of the structure decreases at extreme values of backwardation or contango.

Valuation

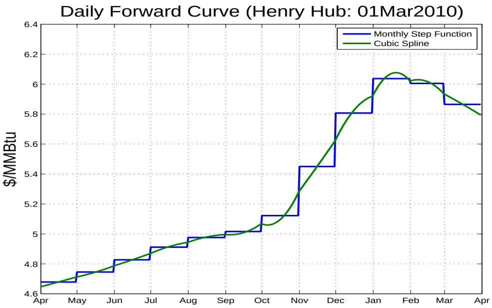
- Results in the one-factor case:



Introduction to Storage Valuation

Daily Forward Curves

- To value natural gas storage for a "high-turn" facility, daily forward curve construction is required.
  - Using a step function for each month results in massive (and unrealistic) value at the monthly boundaries.
  - The daily forward construction must be consistent with traded monthly forwards—as shown in the figure below.



Introduction to Storage Valuation

Daily Forward Curves

- This interpolate was constructed by calculating the cumulative forward value of the commodity:

$$C(\bar{N}_M) = \sum_{m=1}^{M} N_m F_m$$

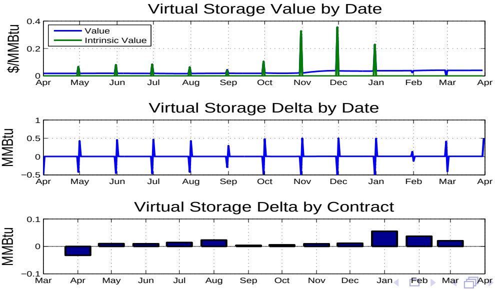
  - N_m is the number of days in month m;
  - F_m is the month forward;
  - $\bar{N}_M = \sum_{m=1}^{M} N_m$ is the number of delivery days through month m.
  - T $\{\bar{N}_m, C(T_m)\}_{m=1}^{M_{\max}}$ was interpolated using a cubic spline to obtain $C(n)$ for all delivery days $n \in [1, N_{\max}]$.
  - $C(\cdot)$ was then differenced to yield a daily forward price $F(0, n)$.
  - Consistency with the monthly forwards is guaranteed:

$$N_m F_m = C(N_m) - C(N_{m-1}) = \sum_{d \in m} F(0, d)$$

Introduction to Storage Valuation

Valuation

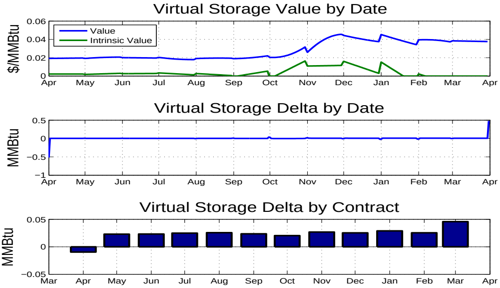
- Working problem: Value a virtual storage contract from 01Apr2010 to 31Mar2011 as of pricing date 01Mar2010.
- As default parameters we will use: $\vec{\beta} = [0.30, 40]$.
- Effect of daily forward curves:
  - This plot shows the value and Δs using the step function.
  - Note the effects of discontinuities in the forward curve of value and exposures at the daily level.



Introduction to Storage Valuation

Valuation

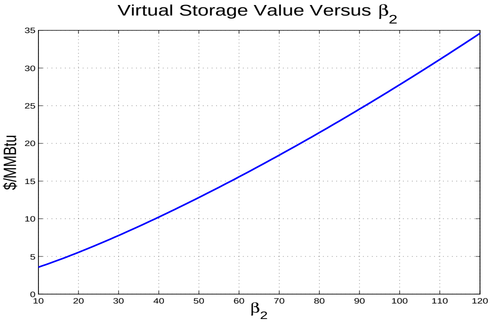
- Effect of daily forward curves:
  - This plot shows the value and Δs using the cubic spline method.
  - The results are more reasonable.
  - However, the fact that a cubic spline has continuity up to only the first derivative is clear in the kinks in value and Δ.



Introduction to Storage Valuation

Valuation

- Effect of mean-reversion rates:
  - This plot shows the value as a function of β_2 holding β_1 constant.
  - The higher the mean-reversion rate the more value due to the higher value of σ_2 required to calibrate to daily vols.



Introduction to Storage Valuation

CSO Subordinators

- Storage with non-trivial constraints requires much more work, typically dynamic programming or approximations thereof.
- CSO's can be used to establish lower bounds on storage value.
  - Denote the present value of the forwards spreads and CSOs as:

$$
\begin{aligned}
S_{i,j} &= e^{-rT_j}F_j - e^{-rT_i}F_i - K_{i,j} \\
C_{i,j} &= \tilde{E}\left[\left(e^{-rT_j}F_j - e^{-rT_i}F_i - K_{i,j}\right)^+\right] \\
P_{i,j} &= \tilde{E}\left[\left(e^{-rT_i}F_i - e^{-rT_j}F_j - K_{j,i}\right)^+\right]
\end{aligned}
$$

    assuming that $i < j$.
  - A lower bound on the value of a storage asset results from the solution of the linear programming problem:

$$
V[S(0), F(0,\cdot)] = \sup_{\substack{\vec{v}\in\mathcal{A} \\ \vec{\alpha},\vec{\beta},\vec{v}\geq 0}} \left(\sum_{1\leq i<j\leq J} \left[\alpha_{i,j}C_{i,j} + \beta_{i,j}P_{i,j}\right] + \sum_{1\leq i<j\leq J} v_{i,j}S_{i,j} \right)
$$

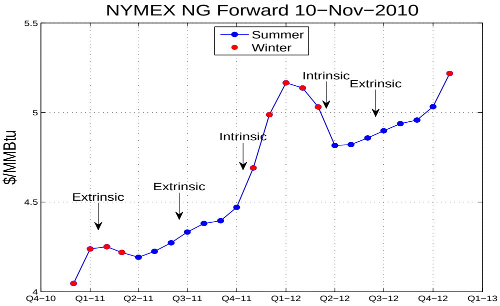
CSO Subordinators

- The annoying part is defining $\mathcal{A}$—the allowed set of spread options.
- While each option may or may not be exercised, it must be assumed to be exercised in order to ensure that the constraints are satisfied under all realizations of forward prices and all possible option exercise events.
- The set of constraints is lengthy but constructable and the optimization problem is amenable to linear programming **once you know the values of the pairwise CSOs**.
- Note that we quietly swept this under the rug in our virtual storage analysis by positing values of $\bar{\beta}$ and assuming uncorrelated BMs and local vol of the form $\sigma(T)$.

Introduction to Storage Valuation

Limited CSO Market Data

- Consider the NG forward curve on 10Nov2010.
  - Seasonality puts the intrinsic value primarily by injecting in the summer and withdrawing in the winter.
  - Extrinsic value is primarily intra-seasonal.
  - This is where correlation information is particularly important.



Introduction to Storage Valuation

Limited CSO Market Data

- Now consider the following broker chat on the same date:
- What's wrong with this picture?

Insert: CSO Broker Chat (10Nov2010)

MARKET LOG FOR MARKET ID:—
11/10/10 8:19 AM v11/f12 -.50 call cso .0825-.095
11/10/10 8:34 AM V/F -50call CSO .075 / .10
11/10/10 8:37 AM V/F -50call CSO .075 @ .09
11/10/10 10:57 AM v11/f12 -.50 call cso @ .0875
11/10/10 10:58 AM v11/f12 -.50 call cso .075-.0875
11/10/10 1:43 PM v11/f12 -.50 call cso 6.5/8.25
11/10/10 1:44 PM v11/f12 -.50 call cso 7/8
11/11/10 8:46 AM v1/f2 -.50 call cso: .07 / .085
11/11/10 1:47 PM v11/f12 -.50 call cso .07-.085
11/11/10 1:52 PM v1/f2 -.50 call cso: .07 / .075
11/11/10 1:52 PM v1/f2 -.50 call cso: .07 / .075
11/11/10 1:59 PM v11/f12 -.50 call cso .07-.075
11/11/10 2:11 PM v11/f12 -.50 call cso .0725-.075
11/12/10 10:16 AM
V/F -1.00/-1.25 1x2 put spr CSO .01/.03 to the 2
V/F -.1.25 put CSO .0675/.075
V/F -1.00 put cso .115/.135
V/F -.50 call CSO .065/.075

Introduction to Storage Valuation

Punchline

- To value physical storage we need to ascribe a value to CSOs that are not commonly traded:
  - Wrong spreads
  - Wrong time-scales (monthly versus daily)
  - Long tenors
- If we can't value CSOs, we certainly can't value storage.
- What do we need to properly value a CSO given limited market data?

Introduction to Storage Valuation

Punchline

- Invoking Magrabe:
  - Term (to expiry) volatilities of the two underlying legs.
    - The second leg involves non-standard expiry—expiration is at least a month before the standard vanilla option.
    - Handling this involves modeling volatility backwardation.
  - Term returns correlation of the underlying legs.
    - This in exceedingly hard (unsolved?) problem.
    - More on this later.
  - Extrapolating the above, which is usually based on monthly products (forwards and options) if not calendar strips, to daily forwards and options for which relatively little trades.

Introduction to Storage Valuation

On Deck

- Term Structure of Volatility and Non-Standard Expiry
- Price Dynamics at the Daily Time-Scale