Lessons From Tolling Deals

Glen Swindle

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Introduction

- Basics of Tolling Deals
- Margrabe Greeks
- Hedging Frictions
- The Vol-Lookup Heuristic
- Conclusion

Basics of Tolling Deals

Deal Structure

- Tolling deal is a derivative rendition of a power generator.
  - Typically the fuel is natural gas.
- The Basic Structure:

$$\tilde{E}\left[\sum_t d(0,t)\max\left(F_P(t,t) - H_* F_G(t,t) - V, 0\right)\right]$$

where:

- t denotes delivery day (this is discrete time).
- F_P denotes the forward (or spot) price of power for a particular delivery bucket (e.g. 5x16) and F_G denotes the price of natural gas; both are typically at liquid pricing hubs.
- H_* is the heatrate (conversion rate between gas and power).
- V is VOM (variable operation and maintenance).

Basics of Tolling Deals

Deal Structure

- The primary purpose of a toll is to annuitize the value of either a soon to be purchased asset or a soon to be built asset in order to facilitate borrowing.
- The typical structure is:
  - Daily day-ahead manual exercise into a standard power buckets.
  - Monthly "capacity payments" (as opposed to upfront premium).
  - Monthly settlement.

  so that the valuation is really:

$$V = \tilde{E}\left[\sum_m d(0, T_m) Q_m \sum_{d \in B(m)} \max\left(F_P(d,d) - H_* F_G(d,d) - V, 0\right)\right]$$

  - $B(m)$ denotes the days in month m with delivery in the relevant power bucket B.

Basics of Tolling Deals

Monthly Toll

- Consider a tolling deal with monthly exercise with the following terms:
  - Pricing date: 11Jan2010
  - Underlyings are PJM Western Hub 5x16 and Henry Hub natural gas.
  - Heatrate $H_* = 11.0$
  - Delivery monthly Jul10
  - Exercise is standard penultimate settlement of NYMEX NG contract.
    - Note: for simplicity we will assume that PJM monthly options also expire '-4b').
  - Notional: 400 MW

Basics of Tolling Deals

Relevant Data

- Remark on Notional:
  - The number of NERC business days in the Jul10 is 21.
  - So the total notional of this toll is $21 \cdot 16 \cdot 400 = 134,400$ MWh.

- The relevant forward prices are:
  - $X_0 \equiv F_P(0, T) = 69.650$
  - $Y_0 \equiv H_* F_G(0, T) = 11.0 \cdot 5.603 = 61.633$

- The discount factor is 0.999 .

- Note: The market heatrate is: $H = \frac{69.650}{5.603} = 12.431.$
  - This is higher than the deal heatrate $H_* = 11.0$
  - The option is in-the-money.

Vol Backwardation:

- We do not have a vol backwardation issue as the respective legs are expiring (by assumption) at their vanilla expiry.

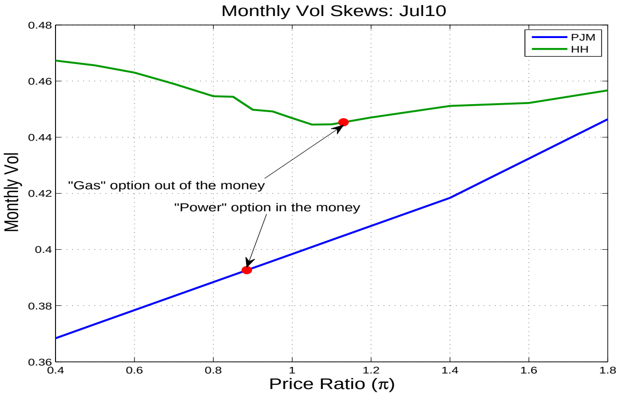
Basics of Tolling Deals

Skew: What vols do we pick?

- There is ambiguity as the vol skews for each underlying are defined in terms of fixed strike options.
- We have floating strikes in the sense that the "other" leg is itself a commodity price.
- A common approach is to use the underlying price of the opposing leg to define moneyness.
- For an option on $F_1 - F_2 - K$ view:
  - F_1 in reference to $F_2 + K$: $\pi_1 \equiv \frac{F(0,T_2)+K}{F(0,T_1)}$.
  - F_2 in reference to $F_1 - K$: $\pi_2 \equiv \frac{F(0,T_1)-K}{F(0,T_2)}$.
- For this problem:
  - $\pi_X \equiv \frac{H_* F_G(0,T)}{F_P(0,T)}$ and $\pi_Y \equiv \frac{F_P(0,T)}{H_* F_G(0,T)}$.
  - In our working problem $\pi_X = 0.885$ and $\pi_Y = 1.130$.
- The results of this vol lookup are: $\sigma_X = 0.393$ and $\sigma_Y = 0.445$.

Basics of Tolling Deals

Skew



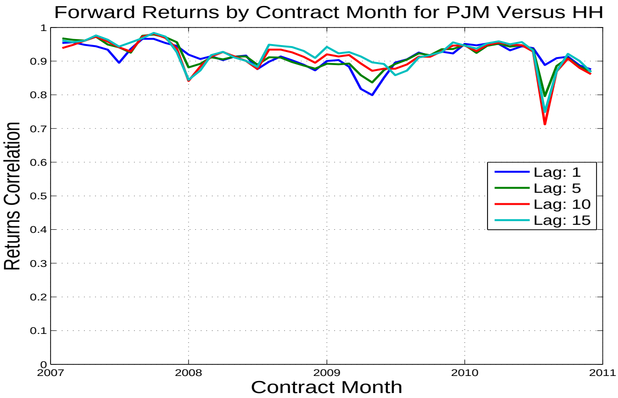
Basics of Tolling Deals

Correlation

- What correlation should we use?
  - Monthly tolling deals are uncommon and there is no broker market from which to calibrate this parameter.
  - The following figure shows the returns correlation by contract month estimated over a 1Y trailing window using several time-scales (lags) for returns:
    - Specifically returns on lag L are defined as $\log\left[\frac{F(d,T)}{F(d-L,T)}\right]$.
    - The purpose of considering returns for $L > 1$ is to both view returns on time scales on which actual hedging activity may occur as well as to smooth out possible anomalies in historical marks.

Basics of Tolling Deals

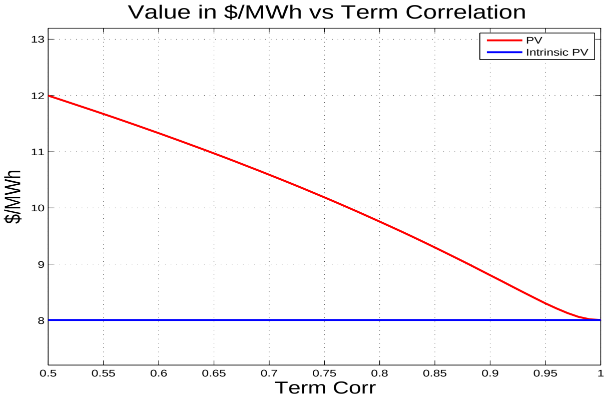
Correlation



Basics of Tolling Deals

Correlation

- The results for the this working problem are shown in the figure below for a range of correlations.



Margrabe Greeks

Delta and Gamma

- Deltas—a sequence of routine calculations yields:

$$\frac{\partial V}{\partial X} = N(d_1) \qquad \frac{\partial V}{\partial Y} = -N(d_2)$$

- Converting to our forward tolling setting yields:

$$\frac{\partial V}{\partial F} = d(\tau)QN(d_1) \qquad \frac{\partial V}{\partial G} = -d(\tau)QH_* N(d_2)$$

- Gamma—similarly:

$$\Gamma = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}} \begin{pmatrix} \frac{1}{F(0,T)} & -\frac{1}{G(0,T)} \\ -\frac{1}{G(0,T)} & \frac{F(0,T)}{G^2(0,T)} \end{pmatrix}$$

- Γ is positive-definite because:

$$\bar{\alpha}^t\Gamma\bar{\alpha} = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}F(0,T)}\left[\alpha_1 - \frac{F(0,T)}{G(0,T)}\alpha_2\right]^2$$

.

- When Δ-hedged, all directions point up.

Margrabe Greeks

Delta and Gamma

- It is useful to diagonalize the matrix to see where the convexity is:
  - The eigenvalues are:

$$\lambda_1 = 0$$
$$\lambda_2 = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}}\left[\frac{1}{F(0,T)} + \frac{F(0,T)}{G(0,T)^2}\right]$$

  - The eigenvector corresponding to $\lambda_1 = 0$ is:

$$\vec{v}_1 = \begin{bmatrix} \frac{F(0,T)}{G(0,T)} \\ 1 \end{bmatrix} \quad (1)$$

  - There is no convexity when market heat rate $H(t,T)$ is constant.
  - The second eigenvector is:

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -\frac{F(0,T)}{G(0,T)} \end{bmatrix} \quad (2)$$

  - Convexity is maximal for price changes in which $\Delta G = -H\Delta F$.
  - This direction is not particularly relevant to empirical natural gas and power dynamics.
  - In this direction, a \$1 increase in *forward* power prices is associated with a roughly \$12 drop in natural gas prices; hardly an expected event.

Margrabe Greeks

Vega

- The valuation above is identical to a call option on X struck at Y_0 with the modification that the term volatility is given by the modified form:

$$\hat{\sigma}^2 = T\left[\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right]$$

- As with a call option, $\frac{\partial V}{\partial \hat{\sigma}}$ is positive:

$$\frac{\partial V}{\partial \hat{\sigma}} = XN'(d_1)\frac{\partial d_1}{\partial \hat{\sigma}} - YN'(d_2)\frac{\partial d_2}{\partial \hat{\sigma}} = XN'(d_1)$$

.

- The chain rule clearly yields:

$$\frac{\partial V}{\partial \sigma_X} = \frac{\partial V}{\partial \hat{\sigma}}\frac{\partial \hat{\sigma}}{\partial \sigma_X}$$

- This means that:

$$\text{sign}\left[\frac{\partial V}{\partial \sigma_X}\right] = \text{sign}\left[\frac{\partial \hat{\sigma}}{\partial \sigma_X}\right] = \text{sign}\left[\sigma_X - \rho\sigma_Y\right]$$

Margrabe Greeks

Vega

- By symmetry:
$$\operatorname{sign}\left[\frac{\partial V}{\partial \sigma_Y}\right] = \operatorname{sign}\left[\sigma_Y - \rho\sigma_X\right]$$
- The implication is that vega with respect to one of the underlyings will be negative if: $\min\left[\frac{\sigma_X}{\sigma_Y}, \frac{\sigma_Y}{\sigma_X}\right] < \rho$.
- As we know from our working example, this situation is not uncommon as σ_X and σ_Y are often of comparable magnitude and ρ is often very close to unity.
- The intuition is simple: if the correlation of the two assets is high and if $\sigma_X > \sigma_Y$ then any increase in σ_Y "chews into" the volatility of the spread $\hat{\sigma}$.

Margrabe Greeks

Vega

- By implicit differentiation:

$$\frac{\partial \hat{\sigma}}{\partial \sigma_X} = \frac{T^{\frac{1}{2}}\left(\sigma_X - \rho\sigma_Y\right)}{\left(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right)^{\frac{1}{2}}}$$

  with a symmetric result for $\frac{\partial \hat{\sigma}}{\partial \sigma_Y}$.

- Therefore:

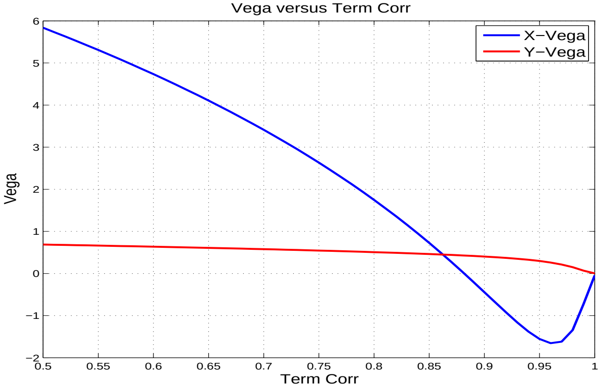
$$\frac{\partial V}{\partial \sigma_X} = XN'(d_1)\frac{T^{\frac{1}{2}}\left(\sigma_X - \rho\sigma_Y\right)}{\left(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right)^{\frac{1}{2}}}$$

  which scales as $T^{\frac{1}{2}}$.

Margrabe Greeks

Vega

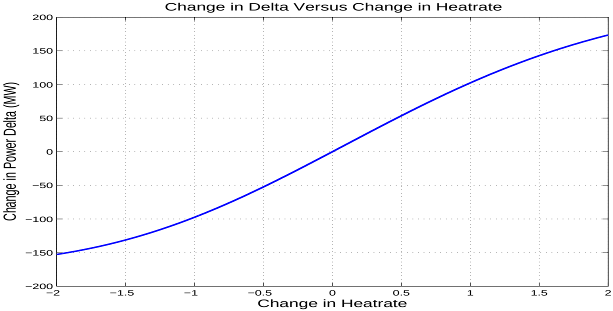
- In our working problem the correlation threshold for negative vega is $\frac{\sigma_X}{\sigma_Y} = 0.882$.
- We expect X-vega to be negative.
- The following plot shows both vegas as a function of correlation.



Hedging Frictions

Block-Size Impact on Hedging

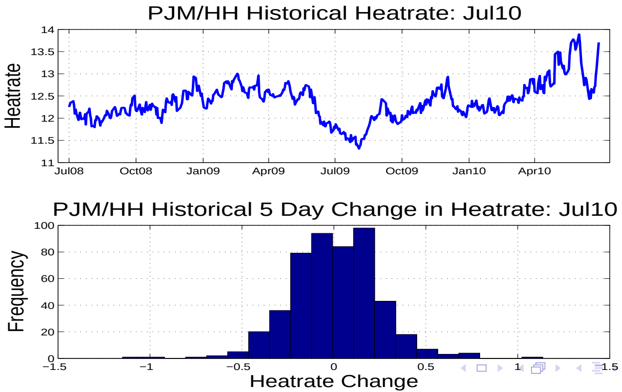
- This has implications for hedging.
  - The fact that you can only trade 50MW limits your ability to capture the modeled extrinsic value.
  - This limitation is an issue in all markets, but particularly so for power and natural gas.
  - The following plot shows the change in delta (converted to MW) as a function of a change in heatrate on the trade date.



Hedging Frictions

Block-Size Impact on Hedging

- How much do heatrates actually move?
  - The following plot shows heatrate history for the Jul10 contract as well as the distribution of 5 day heatrate changes.
  - The conclusion is that delta hedging a 400 MW toll is not going to be a very fruitful enterprise.



Hedging Frictions

Impact of Strips

- Most tolls trade for tenors of 5-7 years.
  - This is well outside the liquidity window at which individual contract months trade.
  - The impact on hedge performance can be non-trivial—but has yet to estimated rigorously.
- In what follows we will switch to the following toll:
  - Pricing date: 11Jan2010
  - Underlyings are PJM Western Hub 5x16 and Henry Hub natural gas.
  - Heatrate: 8.0
  - Delivery period: 01Jan11 to 31Dec11
  - Standard exercise: '-1b'.
  - Notional: 400 .
- Note: A 2-factor Gaussian exponential framework was used for each with correlations specified to be roughly consistent with implied correlations.

Hedging Frictions

Digression on Multi-Factor Valuation

- In the absence of quoted markets for heatrate options, one might consider using estimated correlations as a benchmark.
- In the two-factor framework:

$$\frac{dF_k(t,T)}{F_k(t,T)} = \sum_{j=1}^{2} \left[\int_0^t \sigma_j^{(k)}(T) e^{-\beta_j (T-t)} dB_j^{(k)}(s) \right]$$

  where

  - $k = 1$ corresponds to power.
  - $k = 2$ corresponds to natural gas.

- Factor correlations:
  - $\mathrm{corr}\left[dB_1^{(1)}, dB_1^{(2)}\right] = \rho_{\mathrm{long}}$.
  - $\mathrm{corr}\left[dB_2^{(1)}, dB_2^{(2)}\right] = \rho_{\mathrm{short}}(t)$.

Hedging Frictions

Digression on Multi-Factor Valuation

- In this modeling framework each of the two underlying commodities is log-normally distributed:
  - Valuation eventually leads to Margrabe/quadrature.
  - The term correlation can be explicitly calculated analytically directly from the two-factor diffusions.

- In the "low-budget" model:
  - Each underlying has a log-normal distribution with volatilities $\bar{\sigma}_{D,k}$ which satisfies:

$$\bar{\sigma}^2_{D,k} T_D = \bar{\sigma}^2_{M,k} T_M + \zeta_k^2$$

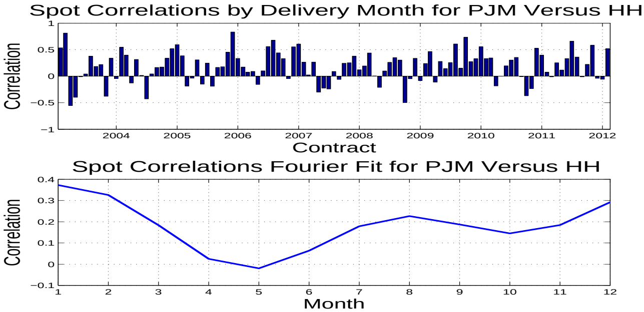
  - The term-correlation is given by:

$$\rho_{\mathrm{term}}\left(T_D\right) = \frac{\bar{\sigma}_{M,1}\bar{\sigma}_{M,2} T_M \rho_L + \zeta_1\zeta_2\rho_S}{\bar{\sigma}_{D,1}\bar{\sigma}_{D,2} T_D}$$

Hedging Frictions

Digression on Multi-Factor Valuation

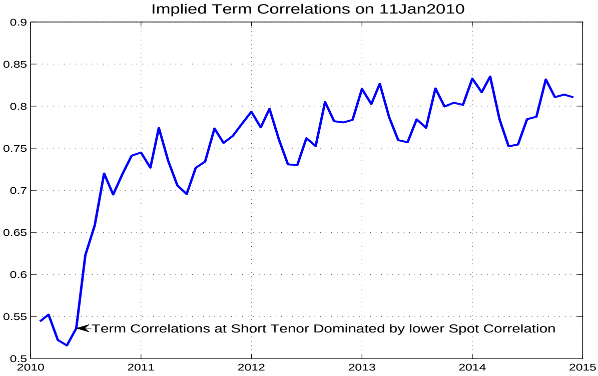
- Note the seasonality:
  - Higher correlation in winter months due to gas price spikes being the driver of power prices.
  - Weather drives summer price dynamics, independently of fuel prices.



Hedging Frictions

Digression on Multi-Factor Valuation

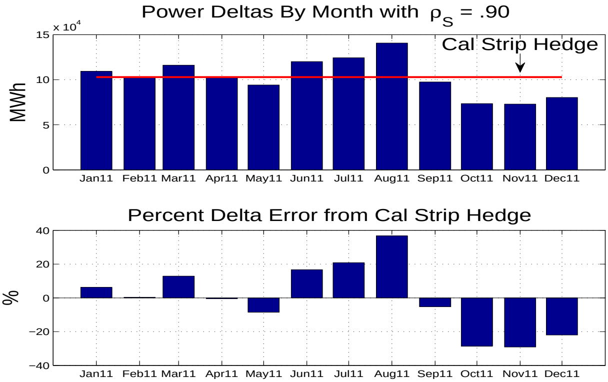
- Using ρ_S from statistical estimate yields the following term structure of correlation.
- Trading activity suggests implied values of ρ_S that are *much* higher.



Hedging Frictions

Impact of Strips

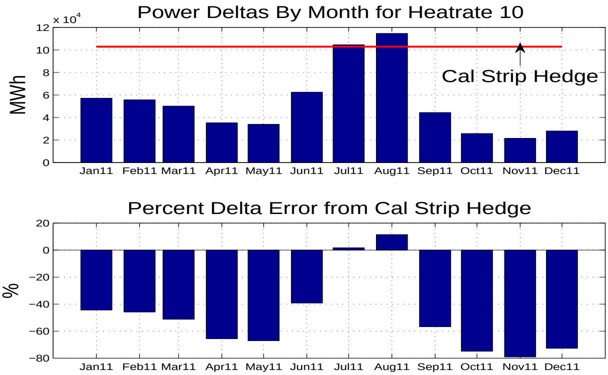
- The following figure shows deltas by month as well as the total cal strip delta, both in absolute terms as as a percentage difference for our toll.
- Note the nontrivial variation from the cal strip quantity.



Hedging Frictions

Impact of Strips

- The problem is exacerbated at higher heatrates, which "pushes" the delta into fewer months making the cal strip instrument an even more blunt instrument.
- The following figure shows the same results for an $H_* = 10$ heatrate.



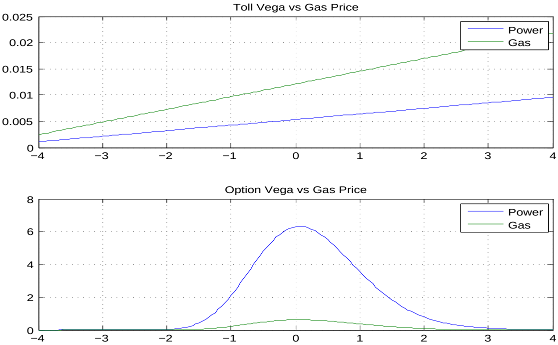
Hedging Frictions

Mismatch in Greek Behavior

- Hedging gamma and vega exposure for tolls is nontrivial:
  - Since vanilla options have a single expiration convention, with expiry near delivery, you have to choose between vega and gamma hedging.
  - Typically you choose to control gamma at shorter tenors and vega and longer tenors.
- The behavior of vega arising from tolling structures is different than that of the vanilla products.
  - The following figures show vega for a sample 8HR toll with tenor 1Y.
  - The underlying prices were \$50/MWh and \$5/MMBtu.
  - The vols were .50 for each leg and correlation was set at .90.

Hedging Frictions

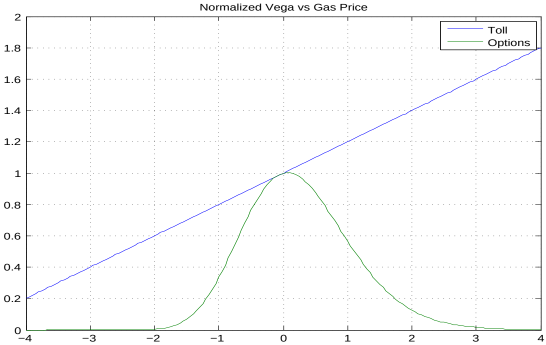
Mismatch in Greek Behavior



- The top figure shows the power and gas vegas for the toll across a range of gas prices assuming that the power price remains at the market heatrate (10).
- The lower figure shows the same for ATM vanilla calls.

Hedging Frictions

Mismatch in Greek Behavior



- The following figure shows the vegas normalized by their respective initial values.
- Due to the symmetry of the market data there is no distinction between power and gas from this perspective.
- Note the meaningfully different behavior both in slope and in magnitude.
- Costly rebalancings of vega hedges would be required to maintain neutrality.

Hedging Frictions

Spurious Risks

- Use of this approach results in exposures to volatility and correlations that can large and unmanagable.
- Are these induced by the choice of model?
- Rephrased:
  - Under this modeling paradigm are changes in volatilities and correlations (if liquidity permitted observation and calibration) off-setting?
  - Value is very dependent on the spot volatility which is driven by the spread between $\bar{\sigma}_D$ and $\bar{\sigma}_M$. Is this real?
  - The absence implied correlation data of a quality analogous to implied vols renders this unanswerable.
- The hedging program that follows is highly questionable.

Hedging Frictions

Spurious Risks

- Vega exposure is not confined to $\bar{\sigma}_D$ for the two commodities.
- The resulting vegas with respect to $\bar{\sigma}_M$ and $\bar{\sigma}_D$ for each of the commodities is largely driven by the implicit correlation effect.

$$\frac{\partial V}{\partial \bar{\sigma}_D} = \frac{\partial V}{\partial \bar{\sigma}_D}|_\rho + \frac{\partial V}{\partial \rho}|_{\bar{\sigma}_D} \frac{\partial \rho}{\partial \bar{\sigma}_D}$$

- The term correlation effect for vega hedging results in substantial positions:
  - Long/short or (short/long) positions in monthly/daily vega depending on relative values of the various component volatilities.

Hedging Frictions

Spurious Risks

- Note that the spread vol that enters Margrabe is:

$$\hat{\sigma}^2 = \left[\bar{\sigma}_{M,1}^2 + \bar{\sigma}_{M,2}^2 - 2\rho_L \bar{\sigma}_{M,1}\bar{\sigma}_{M,2}\right] T_M + \left[\zeta_1^2 + \zeta_2^2 - 2\rho_S \zeta_1 \zeta_2\right]$$

  from which we see that:

$$\frac{1}{2}\hat{\sigma}\frac{\partial \hat{\sigma}}{\partial \bar{\sigma}_{M,1}} = \left[\bar{\sigma}_{M,1} - \rho_L \bar{\sigma}_{M,2}\right] T_M + \frac{\partial \zeta_1}{\partial \bar{\sigma}_{M,1}} \left[\zeta_1 - \zeta_2 \rho_S\right]$$

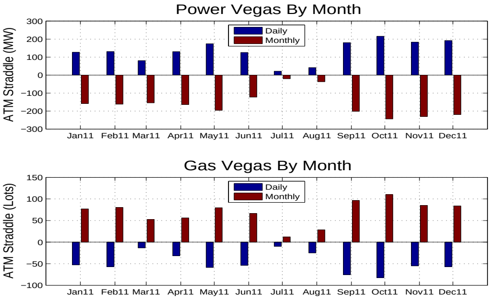
- The sign of the first term on the RHS is exactly like the monthly vega terms in the previous monthly example.

- We know that $\frac{\partial \zeta_1}{\partial \bar{\sigma}_{M,1}} < 0$ and $\frac{\partial \zeta_1}{\partial \bar{\sigma}_{D,1}} > 0$ and similarly for all other permutations of vols and commodity leg.

Hedging Frictions

Spurious Risks

- The figure below shows initial vega exposures in our example.
  - Implied vol hedging strategy involves substantial volume in monthly and daily options in both commodities.
  - Moreover, these hedges would also have to be unwound $t \uparrow T$.



Vol-Lookup Approximation

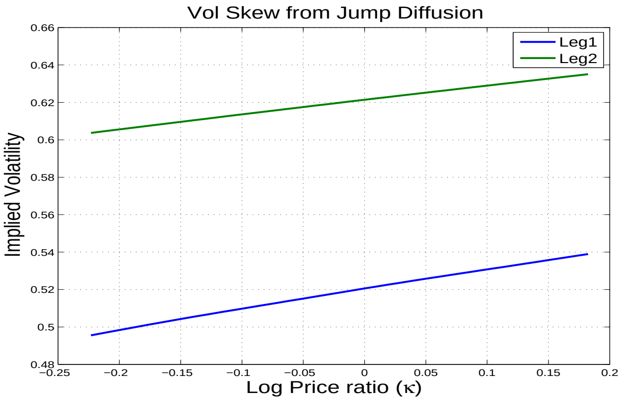
Simple Experiment

- How much error is sustained in the vol-lookup approximation.
- The following analysis considers a reasonable setup:
  - A spread option with tenor of $\frac{1}{4}$ of a year.
  - Power and gas forwards of \$100 and \$10 respectively
  - Heatrates H_* are varied on the interval $[8, 12]$.
- The procedure:
  - N=100,000 standard i.i.d normal deviates $\bar{Z}_n$ where each $\bar{Z}_n \in \Re^2$
  - Poisson jumps with arrival rate 8/year and size 2 were added to $Z_{n,1}$.
  - The $\{\bar{Z}_n\}_{n=1}^N$ were normalized to unit standard deviation and transformed to yield $\bar{X}_n$ with correlation .90.
  - These were normalized to have a standard deviation corresponding to implied vols of .50 and .60 respectively.

Vol-Lookup Approximation

Simple Experiment

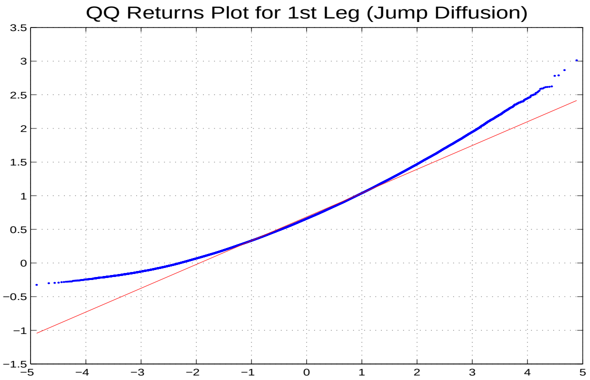
- The following plot shows the skew for the two legs.



Vol-Lookup Approximation

Simple Experiment

- The qq-plot of the returns is shown below.



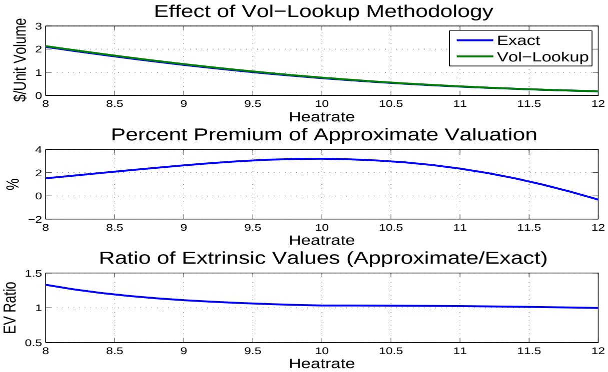
Vol-Lookup Approximation

Simple Experiment

- The following plot shows the results comparing exact (simulation) valuation to the vol-lookup methods as a function of the deal heatrate.
  - The Monte-Carlo error is typically well under $0.10.
  - The vol-lookup approximation is roughly 2% higher at low strikes.
  - The third plot shows the ratios of extrinsic values which is more ominous, with low heatrates seeing ratios above 1.5.

Vol-Lookup Approximation

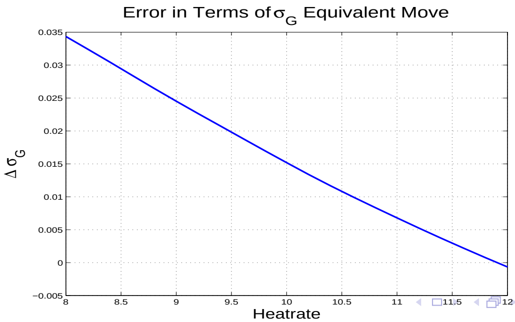
Simple Experiment



Vol-Lookup Approximation

Simple Experiment

- Vol Perspective:
  - Converting this difference in extrinsic value into a σ_G equivalent move by dividing the difference by the G-vega yields the following.
  - The errors can be meaningfully outside of the volatility bid-offer.
  - This has been a subject of some investigation (see C. Alexander and A Venkatramanan 2011).
  - However, a rigorous and efficient methods for bounding this error remain undeveloped.



Conclusions

- Multi-factor models applied to daily tolls in the risk-neutral setting have issues:
  - Spurious risk due to spot correlation effects.
  - Hedging programs that are difficult to affect.
  - Dependence on volatilities and correlations that are often not traded.
- This has spawned interest in alternatives:
  - Econometric models:
    - Calibrated to historical price behavior
    - Ostensibly more realistic price distributions.
    - Valuation can yield estimates of hedging slippage.
    - Difficult to embed anticipated systemic changes.
  - Structural models:
    - Intended to accommodate anticipated systemic changes.
    - Challenging to implement—still a work in progress.

On Deck

- Variable Quantity Swaps and Econometric Models